Electron in metal

Drude model 1900. (Paul Drude)

$q_N = eZ_A$

$q_{core} = -eZ_C$

$q_V = -eZ_v$

Ion: N + core

→ free valence electron in the interstitial holds the ions together

Recall Bonding : Ionic — Covalent — Metallic

Ionic: $Na^+ Cl^-$

Covalent: H·· C ··H (with H above and below C)

Metallic: Electron is shared between many (all) ions → covalent bond with ∞ center.

Why bonds are formed : Atoms want to have spherical charge density to minimize mutual Coulomb repulsion.

Spherical charge density : Completely filled subshell.

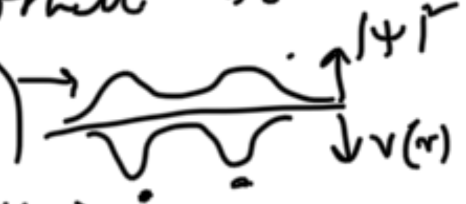
1s, 2s 2p, 3s 3p 3d, ...

Ex

$Na : n_v = 1$; $Cl : n_v = 7$ → $Na^+ Cl^-$: both have complete subshell

$H : n_v = 1$; $H : n_v = 1$ → H··H : " share 2 e → complete subshell $1s^2$

covalent bond $\psi = \frac{1}{\sqrt{2}} (\phi_1(r) + \phi_2(r))$



sharing becomes possible because bonding orbital ψ lowers KE due to hopping (tunnelling).

However 2 Na atoms with $n_v = 1$ → would like to share but too bulky to come close!

Solution ⇒ collective sharing: metallic

Drude applied Boltzmann kinetic theory (BKT) of gases to electrons in metals.

without any external field : electron move in st line without any influence of other electrons → independent electron approximation. (works well).

BKT : electron are colliding with electron and ions with an average interval τ
primarily (Drude)

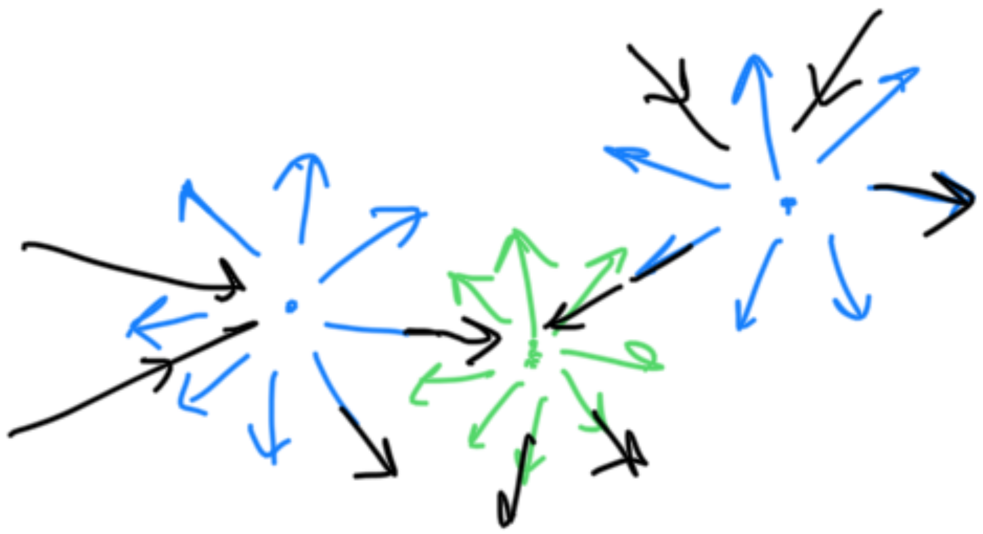
Each collision : electron looses memory, resets velocity according to local thermal environment and emerges randomly directed.

$\Rightarrow$ 

Any two possible directions randomly chosen

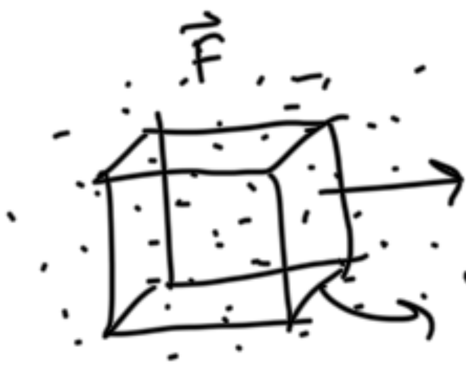
$|p| \sim \sqrt{\frac{3k_BT}{m}}$ (too crude at low temp)

No net momentum at $|\vec{F}| = 0$.



So each time they collide they thermalize

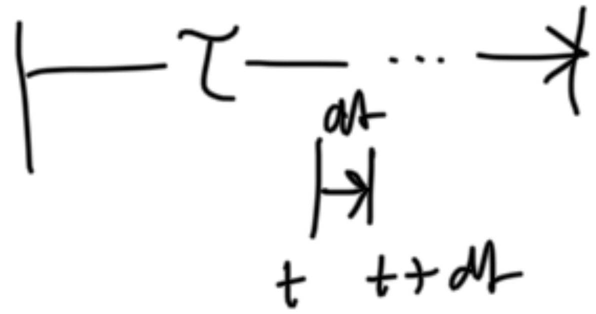
However, with $|\vec{F}| > 0$, $\vec{F} \parallel \hat{x}$



net momentum $N\vec{p}(t)$ along $\hat{x}$ $\Rightarrow$ current

volume containing N electron at time t with average momentum $\vec{p}(t)$

Drude Theory:



Let average momentum of electron at t is $\vec{p}(t)$

Total momentum of N electron at t : $N\vec{p}(t)$

Estimated # of collision : $dt = N\,dt/\tau$

" # " electron survive $dt = N(1 - dt/\tau)$

Total momentum of N electrons at $(t+dt) = N\left(1 - \frac{dt}{\tau}\right)\left(\vec{p}(t) + \vec{F}\,dt\right) + \frac{N\,dt}{\tau}\left(\vec{F}\frac{dt}{2}\right)$ ↳ Assuming collission occuring uniformly throughout dt

$\therefore$ $\approx N\vec{p}(t) + N\vec{F}dt - \frac{N\,dt}{\tau}\vec{p}(t) + O(dt^2)$

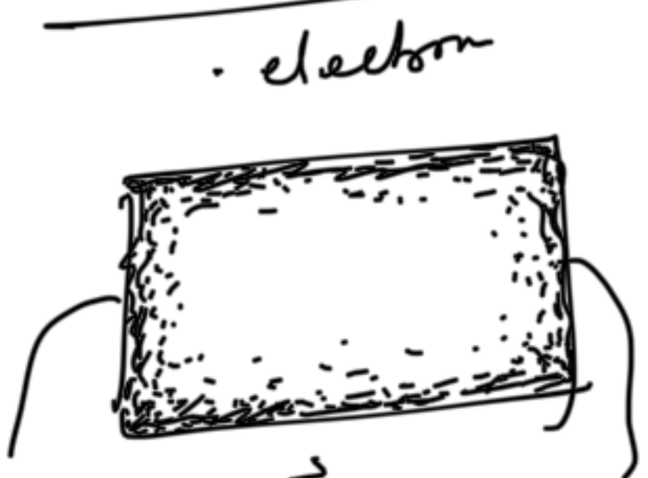
(retaining only order of dt)

$\therefore$ Average momentum of electron at $(t+dt)$ is

$$\vec{p}(t+dt) = \vec{p}(t) + \vec{F}dt - \vec{p}(t)\frac{dt}{\tau} \Rightarrow \boxed{\frac{d\vec{p}}{dt} = \vec{F} - \frac{\vec{p}(t)}{\tau}}$$

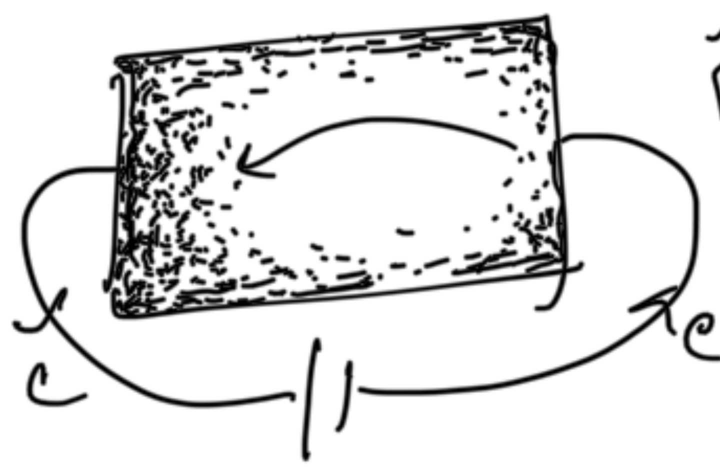
$\therefore$ If $\vec{F} = 0$, $\vec{p}(t+\Delta t) = \vec{p}(t)e^{-\Delta t/\tau}$ → survived equation originating from thermalization at an average interval τ.

DC Flow



Switch on $\vec{E}$

Connect battery



Takes finite time for the concentration gradient to stabilize

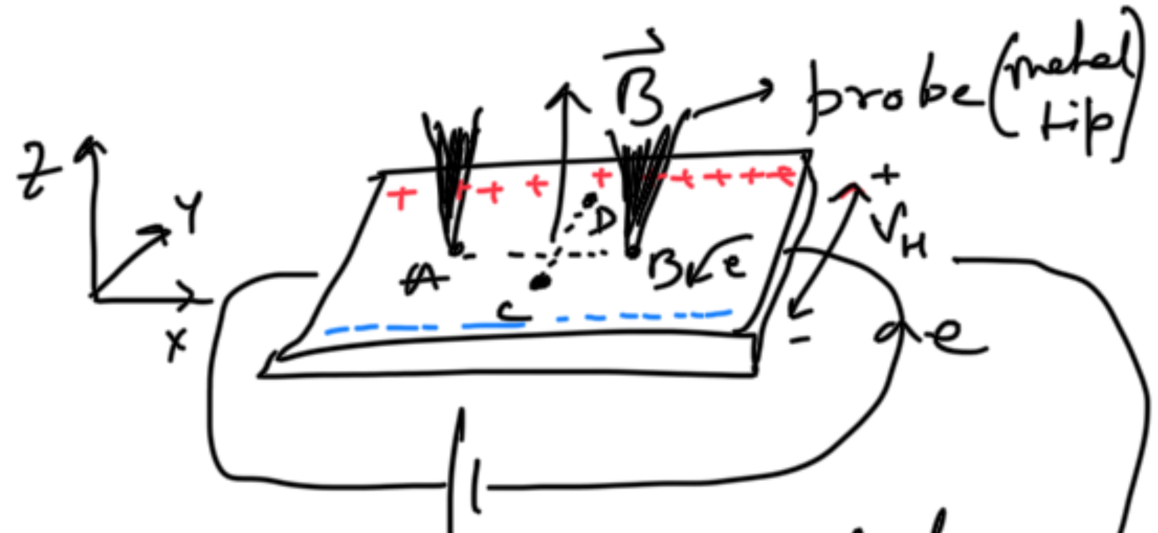
Steady state: $\frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \tau\vec{F} \Rightarrow \vec{v} = -\frac{e\vec{E}\tau}{m}$; $\vec{F} = -e\vec{E}$

$$\Rightarrow \vec{j} = -en\vec{v} = +e^2 n\tau\vec{E}/m \Rightarrow \sigma = |\vec{j}|/|\vec{E}| = \boxed{\frac{e^2 n\tau}{m} = \sigma_{DC}}$$

In presence of magnetic field:

$$\frac{d\vec{p}}{dt} = -e\left[\vec{E} + \vec{v}\times\vec{B}\right] - \frac{\vec{p}}{\tau}$$

$$= -e\vec{E} + \frac{\vec{j}}{n}\times B + \frac{m\vec{j}}{ne}\frac{1}{\tau}$$



Current i measured from A to B and C to D.

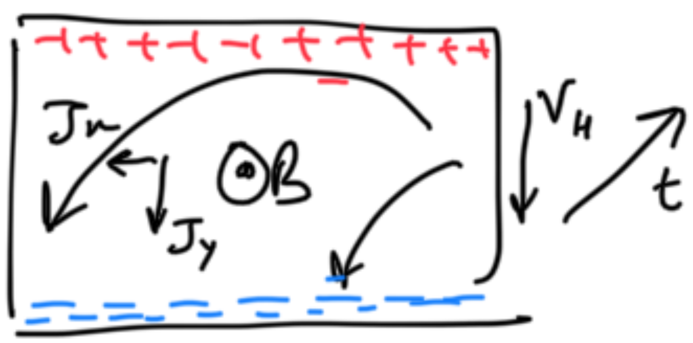
Steady state:

$$0 = -e\left[E_x\hat{x} + E_y\hat{y}\right] + \frac{1}{n}\left[J_x\hat{x} + J_y\hat{y}\right]\times(B\hat{z}) + \frac{m}{ne\tau}(J_x\hat{x} + J_y\hat{y})$$

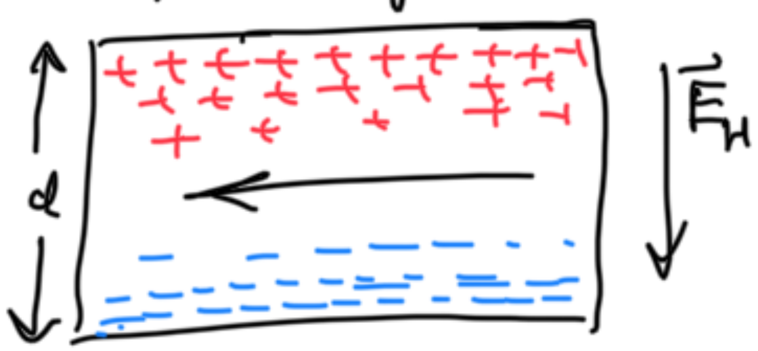
↳ source: V_H caused by deflection of electron due to $\vec{B}$

At steady state J_y should be zero and J_x constant. $\Rightarrow \frac{\partial J}{\partial t} = 0 \Rightarrow \frac{\partial p}{\partial t} = 0$; $\because j = -\frac{enp}{m}$

Transient



steady state



with time $\vec{E}_H$ builds up and balances the Lorentz force.

$$\vec{E}_H = \frac{\vec{j}\times\vec{B}}{n} \rightarrow \text{No deflection}$$

$$V_H = d\,\frac{|\vec{j}\times\vec{B}|}{n}$$

$$\Rightarrow E_x\hat{x} + E_y\hat{y} = \frac{B}{ne}\left[J_x(-\hat{y}) + J_y\hat{x}\right] + \frac{m}{ne^2\tau}(J_x\hat{x} + J_y\hat{y})$$

Along $\hat{x}$: $E_x = \frac{B}{ne}J_y + \frac{m}{ne^2\tau}J_x$

Along $\hat{y}$: $E_y = -\frac{B}{ne}J_x + \frac{m}{ne^2\tau}J_y$

$$\rightarrow \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \frac{m}{ne^2\tau} & \frac{B}{ne} \\ -\frac{B}{ne} & \frac{m}{ne^2\tau} \end{bmatrix}\begin{bmatrix} J_x \\ J_y \end{bmatrix}$$

Now since $J_y = 0$ at steady state: $E_y = -\frac{B}{ne}J_x$

Define resistivity tensor $\underline{\underline{\rho}} = \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{bmatrix}$ in 2D: $\underline{E} = \underline{\underline{\rho}}\ \underline{J}$

Hall resistivity: $\rho_H = \frac{\partial E_y}{\partial J_x} = -\frac{B}{ne}$

Hall coeff: (Drude): $R_H = \frac{E_y}{J_x B} = -\frac{1}{ne}$ $\rightarrow$ estimate of e or n.